SUPPLEMENTARY APPENDIX

Strategy-proof and Efficient Mediation: An Ordinal Market Design Approach

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Model Illustrations and Real-life Mediation/Negotiation Cases

We provide several mediation and negotiation cases and briefly discuss how they fit our formal model.[1]

University of Cambridgeville Expansion Dispute

This is an example for joint issues with discrete set of alternatives. University of Cambridgeville (UC) is a private university located at Cambridge, Massachusetts. The university received a generous donation from several big tech companies. UC plans to expand the department of nanotechnology and open a new nanotechnology center. As a part of this expansion plan, UC aims to construct a new building with a few dozen state-of-the-art labs, classrooms, and offices. The new center is expected to accommodate several hundred undergraduate and graduate students, faculty, and staff.

The mayor is very happy with this expansion plan since the center will be an incubator for new projects, patents and startup companies, which means good news for the city's tax revenue. The center is also likely to attract other big companies to Cambridgeville in the long run. The mayor has already started the process with the zoning department for allowing a 10 story building in the campus of UC.

UC has announced their expansion plan a year ago and received a strong pushback from the community. Community members are already sick and tired of parking problems and constantly increasing rents and housing cost in the last 10 years. Since the University's announcement of the expansion plan, rents have already increased 20% in a singe year. Community members are afraid that rents will skyrocket with the flow of startup companies. Finding parking on the street is already a big headache for the neighbors and they sometimes spend hours to find a parking spot. This expansion will likely make parking and traffic unbearable for many. The neighbors plan to put pressure on the city council and take the case to the court as a last resort. Although they know the issue will likely be resolved in favor of the university in the end, they hope to make their case and postpone the process to discourage the university and all other like-minded companies in the future.

The case goes to mediation, where representatives of the UC are on one side and representatives of the neighborhood association are on the other side. During mediation,

[1] This collection of cases are borrowed from Program on Negotiation (PON) at Harvard Law School and Erwin Mediation Services (http://www.erwin-mediation.ie/case-studies/partnership-mediation/). For brevity, we shorten/paraphrase the cases.

UC declares that part of their expansion plan includes building a parking garage and another building for housing to accommodate some new faculty members and graduate students. Discussions evolve around whether the university can accept a costlier plan to build a three or four-story parking garage rather than a one-story one and increase the number of housing units.

Fitting the model: In this dispute, the main issue i.e., X, is whether the UC will continue with the current expansion plan or not; and if it does, what would be the size of the new center, e.g., the center could be a small one with a four-story building, or a medium size one with a six-story building, or a large one with ten floors. The two sides clearly have diametrically opposed preferences over this issue. The issue also has uncertain gains from mediation since it is not a priori clear whether it will be possible for the expansion plan to materialize in some form, e.g., whether a mutually acceptable resolution exists. The mediation talks reveal a second issue, i.e., Y, which is the size of the parking garage and the number of housing units. UC intends to build a parking garage and new housing units although it prefers them to be at minimum levels, whereas the neighborhood prefers the opposite. The issue has certain gains from mediation since a new parking garage and housing units are certain to be constructed and the representatives of both sides know that there is a zone for negotiation. The two issues are joint because if the new center is not built, there would no longer be any need for a new parking garage or housing units.

A Business Negotiation Case Study: Ending the NHL Lockout

This is an example for separate issues with a continuum of alternatives. In July 2012, National Hockey League (NHL) opened negotiations for a new collective-bargaining agreement with an aggressive proposal to reduce players' percentage of hockey-related revenue from 57% to 43%, among other demands. After waiting a month, the NHL Players' Association (NHLPA) put forth an offer that separated player salaries from league revenue, slowing the growth of player salaries, and dividing revenues saved among financially struggling teams.

The league lockout came after the September 15, 2012 expiration date of the parties' existing contract came and went. Weeks of cancelled games turned into months. A breakthrough came when federal mediator Scot L. Beckenbaugh entered the picture, according to USA Today. When face-to-face negotiations got heated, Beckenbaugh separated the two sides and engaged in shuttle diplomacy, visiting each side in turn to identify issues where they were willing to be. The final deal hinged on the issue of player pensions. The agreement allowed NHL players, whose careers are often short, to concede on the short-term issue of salary in return for peace of mind regarding their long-term financial future.

Fitting the model: In this negotiation, the main issue is players' percentage of hockey-related revenue. The outside option is impasse (no deal) which negatively affects both parties; however it is unclear how badly the two parties want to make a deal. The issue has uncertain gains from mediation since there may be no mutually acceptable percentage level. How high a players' revenue percentage NHL is willing to accept is privately known, i.e., what percentage level within the interval $[43\%, 57\%]$ is the deal-breaker. So is the least-acceptable percentage level to NHLPA. The second issue is player pensions, on which there is less tension and clear room for negotiation. The outside option in this issue is leaving the pensions as is, but it appears that both parties are open to negotiate a better deal. The second issue exhibits certain gains from mediation. This is an examaple of a negotiation with separate issues since players' percentage from the revenue can be considered independently of the pensions.

Detroit Moves Forward, Thanks to Mediation

This is an example for separate issues with a continuum of alternatives. Detroit navigated the bankruptcy process with less difficulty than had been anticipated by the help of mediation process. The prospect of a long and rancorous court battle was defused when the major players involved, including the city's retired workers and financial creditors, agreed, at the bankruptcy judge's urging, to hash out the terms of a deal in a private mediation process. Months of mediation sessions led to an "exit plan that was more a deal than a court-imposed solution," according to the Times.

In the Detroit mediation, the city's retirees became amenable to agreement thanks to a so-called "grand bargain" in which foundations, the state of Michigan, and the Detroit Institute of the Arts promised to chip in millions to bolster the city's pension system. As for the city's creditors, they had to accept low recovery rates on Detroit's debt during mediation, though some were promised opportunities to make up their losses through future real estate and infrastructure projects in the city.

Fitting the model: This is a complex, multi-party dispute. The main issue is the amount of debt, if any, to be paid out to the creditors, which has uncertain gains from mediation. The outside option is a court-imposed solution. Each party has a private expectation about the outcome of the court's decision. Whether there is room for negotiation hinges on the two sides' expectation and willingness to pay/collect. The second issue is the volume of future real estate infrastructure projects which is used for leveraging the bargaining. The extent of the promises for future projects can be viewed as exhibiting certain gains. Although these are mutually beneficial agreements, the state is forgoing potentially more advantageous deals in the future; thus, contrary to the creditors, it would prefer to make fewer such promises.

Partnership Mediation

This is an example for joint issues with a continuum and discrete set of alternatives. There were three partners in the dental practice. Two had been partners from the beginning and when the practice expanded, they had invited the third person to join them. They came to agreement about the percentage of drawings that would be likely to be earned in each year; the amount that would be paid out automatically each month; the amount that the new partner had to pay to get into the practice; the hours and duty rosters were also agreed. The three dentists would pay for and share the administration staff, the premises, IT and other outgoings. Initially the partnership worked well, and the patients liked the new partner. All went well for some years but then differences started to appear between the new partners and the original partners. Complaints started to be made by the staff that the new partner was being rude, wasn't fulfilling her roster and wasn't keeping up with the administration of the practice which was having an effect on the finances. The partners had tried to sit down together to see if the matter could be resolved but they were not able to make any progress. Then the new partner went on sick leave due to stress. A locum was organized but it was clear that the matter had to be tackled.

Pre-Mediation: The mediator met each of the parties separately so that he could understand what was going on. The original partner said that he had run out of patience with the new partner. She was being abusive to him and wouldn't give a civil answer to any questions. As she wasn't doing her administration, the practice was being put in a difficult situation financially. The new partners' treatment of staff created a risk that a formal complaint would be made, or that someone else would be out on stress leave and the practice couldn't withstand that. The second partner said that he was less involved from the day to day operations. He didn't have much to do with the new partner, but for the sake of the practice and the staff things had to be sorted out. The new partner was equally unhappy. She said that the duty rosters had been misrepresented to her as had the amount of partners' drawings. She wasn't getting the holidays she was entitled to and she wasn't being given the respect she deserved by the staff. She couldn't work with the main partner and she felt that he didn't appreciate her worth. She wasn't coming back to work until they had thrashed things out.

The mediator asked each of the partners to think about what each of them wanted from the mediation—what would their ideal solution be.

Mediation: On the day of the mediation the mediator met the three partners together and each told of their issues. The exchanges were heated, and people were accused of not telling the truth. From the exchanges a number of issues were raised that might form the basis for the partners coming to an agreement. That said, the mediator didn't get a sense that there would be a genuine meeting of minds and he wasn't sure a mediated agreement would last given the level of animosity between them.

The mediator asked to meet the parties separately with the new partner on the one hand the original partners on the other. He met the new partner first and discussed with her how she felt things were going and whether she felt that there could be a workable arrangement with the other partners. The mediator asked if she had considered leaving the practice. She had thought about leaving, but didn't know how it would work nor how much she would be paid to leave. The mediator asked if she wanted him to explore this option with the other partners and she confirmed that he could.

The mediator then talked to the original partners and asked if they had considered the buy-out option. They hadn't thought about it but were willing to do so provided that the "sums" were right. The parties didn't meet together again in the mediation that day, but it was agreed that the parties would seek professional advice on the value of the new partner's practice and when this was to hand the mediation would reconvene.

When the mediation reconvened some weeks later the new partner told the mediator the amount that she wanted to leave the practice and the mediator put that to the others. They didn't agree that figure but made a counteroffer. The offer started to expand to include not only the amount of the buyout but also when the partner would leave; what would be said to the patients; what would be said to the staff; what equipment/items in the practice could be removed by the new partner. The agreement negotiated included precise details in relation to the buyout payment which was to be by way of staged payments. The actual negotiation to arrive at an agreement on all items took many hours but in the end the mediator drew up a legally binding agreement which was signed by all parties.

Fitting the model: In this dispute, the main issue is the buyout price that the new partner needs to be paid to accept to leave the practice. The second issue would be all the other terms that need to be settled if she agrees to leave, e.g., which equipments she can take with her. The outside option for both issues would be the same: the new partner doesn't leave the practice, or the dispute/impasse continues. The issues are clearly joint. The main issue has uncertain gains since mediation would break down if there is no room for negotiating the price, e.g., the maximum buyout price the old partners are willing to offer turns out to be below the minimum price the new partner is willing to accept. Once the main issue is resolved, solving the second issue is of mutual interest, i.e., mutually acceptable resolutions exist and are efficient relative to impasse. For example, what will be said to patients or staff is important for the reputation of all involved parties. Similarly, equipments that are no longer needed by the old partners would be cheaper to acquire by the new partner compared to a new purchase.

Buying a Home and Dealing with Difficult People

This is an example for a joint issues with a continuum and discrete set of alternatives. Imagine that you and your family have moved to a new town. You are living in a month-to-month rental and have finally found the perfect house to buy. Unfortunately, the seller is being unreasonable. The house is on the market for $600,000, but your research, backed up by your broker's opinion, tells you it's overpriced. By your estimate, a fair price would be $500,000, but when you offer that amount, the seller tells you that you are "not even close" and doesn't counter. You think the seller is in denial about the slump in the housing market, which has affected prices in your town quite a bit.

In this circumstance, a "loose" deal structure might provide a way forward. Imagine that you make a slightly higher offer (it is, after all, your "perfect" house) of, say, $525,000. Then give the seller 60 days to keep shopping the house. If a better offer comes in during that time, the seller has the right to walk away by paying you a "breakup fee" say, $25,000 or less.

Although this kind of deal structure isn't very common in the United States, it is actually the norm in the United Kingdom. There, a homeowner can formally accept an offer from one buyer but remain open to competing offers up until the moment of closing. The British even have a word to describe what happens when a third party jumps a deal: gazumping. The buyer can even buy "gazumping insurance" against getting gazumped.

Fitting the model: In the negotiation, the main issue is the house price, where the parties' valuations are their private information. In fact, since a gazumping deal is considered when initial price negotiations fail, the issue is the house price within the *initially disagreed* price range. Then the resolution strategy suggested here is to bring a second issue to the table where parties negotiate about the number of days the house owner can keep the house on the market. (The breakup fee may also be considered as a third issue.) The main issue has uncertain gains since there may not always be a mutually acceptable price regardless of the days the house stays on the market. Conditional on an agreement on the main issue, the seller would prefer to have the house on the market for as long as possible, whereas the buyer would prefer such time to be as short as possible. The popular use of this type of deal in practice suggests that there are often mutually acceptable lengths of time during which the seller can keep the house on the market.

Proof of Theorem 3

Theorem 3 (Visual Characterization). *The following statements are equivalent for the lower half of the mediation rule f, corresponding to type profiles in which a mutually acceptable alternative from the main issue exists;*

(*i*) *f is a logrolling rule.*

(*ii*) *The triangle $\triangle_{m,1}$ has a rectangular partition such that f assigns a unique bundle from the set of logrolling bundles B^t to each rectangle in this partition.*[2]

Proof. We start with $(i) \Rightarrow (ii)$. Let $f_{r_1,r_1} = (x_{r_1}, t(x_{r_1})) \in B^t$ where $x_{r_1} = \max_X \trianglerighteq$ and $1 \leq r_1 \leq m$. Because the mediation rule f always chooses the alternative in the first issue that maximizes $\trianglerighteq$, all the entries on row r_1 to the left of entry f_{r_1,r_1}, all the entries on column r_1 below entry f_{r_1,r_1}, and all the entries in between must fill up with bundle $(x_{r_1}, t(x_{r_1}))$ because x_{r_1} has the highest rank over X. Thus, the rectangle $\square^{r_1}_{m,1}$ fills up with $(x_{r_1}, t(x_{r_1}))$.

Let $\square^{r_1}_{m,1}$ be the first element of the rectangular partition of $\triangle_{m,1}$. Note that, when $m \geq 3$, the so-far-unfilled $\triangle_{m,1} \backslash \square^{r_1}_{m,1}$ consists of at least one triangle (if $r_1 \in \{1, m\}$) and at most two triangles (if $r_1 \notin \{1, m\}$).

[2]More formally, for any $\square$ in the partition of $\triangle_{m,1}$ and any bundles $b, b' \in \square$, $b = b'$; but for any distinct pair $\square, \square'$ in the partition of $\triangle_{m,1}$, $(x, y) \in \square$ and $(x', y') \in \square'$ implies $x \neq x'$ and $y \neq y'$.

Next, take an arbitrary triangle $\triangle_{s,r} \in \triangle_{m,1} \backslash \square^{r_1}_{m,1}$. Note that either $s = r_1$ and $r = 1$, or $s = m$ and $r = r_1 + 1$. Let $f_{r_2,r_2} = (x_{r_2}, t(x_{r_2})) \in B^t$ with $r_2 \neq r_1$ denote the logrolling bundle on the hypotenuse of $\triangle_{s,r}$ that satisfies $x_{r_2} = \underset{X_{sr}}{\boldsymbol{max}} \trianglerighteq$. Once again, starting from the hypotenuse of $\triangle_{s,r}$ all the so-far-unfilled entries on row r_2 to the left of entry f_{r_2,r_2}, all the so-far-unfilled entries on column r_2 below entry f_{r_2,r_2}, and all entries in between must fill up with bundle $(x_{r_2}, t(x_{r_2}))$ because x_{r_2} has the highest rank among X_{sr}. Thus, let $\square^{r_2}_{s,r}$ denote the second element of the rectangular partition of $\triangle_{m,1}$.

Note that the so-far-unfilled set $\triangle_{m,1} \backslash \left\{\square^{r_1}_{m,1} \cup \square^{r_2}_{s,r}\right\}$ consists of at least one triangle. Iterate this reasoning and at each step pick a triangle from the so-far-unfilled subset of $\triangle_{m,1}$ and fill its corresponding rectangle with the bundle whose first component has the highest precedence with respect to $\trianglerighteq$. By the finiteness of the problem, the rectangular partition is obtained in m steps.

Now we show $(ii) \Rightarrow (i)$. Consider a rectangular partition $\mathcal{P}^1$ of $\triangle_{m,1} (\equiv \triangle^1)$. Let $\square^{r_1} \subset \triangle^1$ be the rectangle that includes the entry at the bottom left corner of triangle $\triangle^1$, i.e., $f_{m,1}$. We construct the precedence order $\trianglerighteq$ as follows: Let $f^X_{m,1} = x_{r_1}$ have the higher precedence rank than any other alternative in X, i.e., $x_{r_1} \trianglerighteq x$ for all $x \in X$. Next consider $\triangle^1 \backslash \square^{r_1}$ which has a triangular partition $\mathcal{P}^2$ that consists of at most two triangles.

Take an arbitrary triangle $\triangle^2 \in \mathcal{P}^2$ and let $\square^{r_2} \subset \triangle^2$ denote the rectangle that includes the entry at the bottom left corner of triangle $\triangle^2$, say $(x_{r_2}, t(x_{r_2}))$. Let x_{r_2} have a higher precedence rank than any other alternative in X that appears on the hypotenuse of $\triangle^2$. Namely, if $r_2 < r_1$, then $x_{r_2} \trianglerighteq f^X_{k,k}$ for all $k \in \{1, ..., r_2 - 1, r_2 + 1, ..., r_1 - 1\}$, and if $r_2 > r_1$, then $x_{r_2} \trianglerighteq f^X_{k,k}$ for all $k \in \{r_1 + 1, ..., r_2 - 1, r_2 + 1, ..., m\}$.

Iterate in this fashion by considering an arbitrary triangle from the remaining partition $\triangle^1 \backslash \{\square^{r_1}, \square^{r_2}\}$. At the end of this finite procedure (consisting of exactly m steps), we obtain a transitive, antisymmetric but possibly incomplete strict precedence order $\trianglerighteq$ on B^t. Moreover, by construction we have $f_{\ell,j} = \left(x^*_{X_{j\ell}}, t\left(x^*_{X_{j\ell}}\right)\right)$ where $x^*_{X_{j\ell}} = \underset{X_{j\ell}}{\boldsymbol{max}} \trianglerighteq$ for all $1 \leq j \leq \ell \leq m$. This completes the proof. $\square$

Proof of Theorem 4

Theorem 4. *A mediation rule minimizes rank variance within the class of logrolling rules if and only if it is a constrained shortlisting rule.*

Proof. Clearly, a CS rule belongs to the logrolling rules family. Fix the set of logrolling bundles $B^t = \{(x, t(x)) | x \in X\}$ and the family of logrolling rules whose range is $B^t \cup \{(o_X, y)\}$ for some $y \in Y$. Let $b_j = (x_j, t(x_j)) \in B^t$ denote a logrolling bundle. To see that the rank variance of a CS rule is lower than any other member of the logrolling rules family, we simply consider two cases about the number of possible alternatives. First, when m is odd, $var(b_k) = (m+1)^2$. For any $b_{k-j}, b_{k+j} \in B^t$ with $j < k$, we have $var(b_{k-j}) = var(b_{k+j}) = 2(\frac{(m+1)}{2} - j)^2 + 2(\frac{(m+1)}{2} + j)^2 = (m+1)^2 + 4j^2$. Thus, $var(b_k) < var(b)$ for any $b \in B^t \setminus \{b_k\}$.

Since any member of the logrolling rules family must pick an element of B^t whenever there is a mutually acceptable alternative in issue X (by Theorem 1), minimization of rank variance requires that $x_k \trianglerighteq x$ for any $x \in X$. Also observe that $var(b_k) < var(b_{k-1}) < \ldots < var(b_1)$ and $var(b_k) < var(b_{k+1}) < \ldots < var(b_m)$. Thus, minimization of rank variance subsequently requires that $x_{k-1} \trianglerighteq \ldots \trianglerighteq x_1$ and $x_{k+1} \trianglerighteq \ldots \trianglerighteq x_m$. By Theorem 1 the outcome for issue X is fixed to o_X whenever there is no mutually acceptable alternative in this issue. Therefore, (o_X, y_k) is the rank-variance-minimizing bundle. Note that when m is odd, rank variance of the unique CS rule is strictly less than any other member of the logrolling rules family.

On the other hand, when m is even, $var(b_{\overline{k}}) = var(b_{\underline{k}}) = \frac{1}{2}(m^2 + (m+2)^2)$. For any $b_{\underline{k}-j}, b_{\overline{k}+j} \in B^t$ with $j < k$, we have $var(b_{\underline{k}-j}) = var(b_{\overline{k}+j}) = 2(\frac{m}{2} - j)^2 + 2(\frac{(m+2)}{2} + j)^2 = \frac{1}{2}(m^2 + (m+2)^2) + 4j^2$. Hence, $var(b_{\overline{k}}) = var(b_{\underline{k}}) < var(b)$ for any $b \in B^t \backslash \{b_{\overline{k}}, b_{\underline{k}}\}$. Note that we also have $var(b_{\underline{k}}) = var(b_{\overline{k}}) < var(b_{\underline{k}-1}) < \ldots < var(b_1)$ and $var(b_{\underline{k}}) = var(b_{\overline{k}}) < (b_{\overline{k}+1}) < \ldots < var(b_m)$. Then, minimization of rank variance subsequently requires that either $x_{\overline{k}} \trianglerighteq x_{\underline{k}}$ or $x_{\underline{k}} \trianglerighteq x_{\overline{k}}$ together with $x_{\underline{k}-1} \trianglerighteq \ldots \trianglerighteq x_1$ and $x_{\overline{k}+1} \trianglerighteq \ldots \trianglerighteq x_m$. By Theorem 1 the outcome for issue X is o_X and both $(o_X, y_{\overline{k}})$ and $(o_X, y_{\underline{k}})$ are rank-variance-minimizing bundles. Consequently, any one of the four types of CS rules are rank variance minimizing. Note that when m is even, rank variance of a CS rule is weakly less than any other member of the logrolling rules family. □

Symmetric Treatment of the Outside Options

We now consider a relaxation of the assumption that $y \; \theta_i^Y \, o_Y$ for all $i \in \mathrm{N}$ and $y \in Y$. Namely, we suppose that the ranking of each outside option is negotiators' private informations. Let $\Theta_i = \Theta_i^X \times \Theta_i^Y$ denote the set of all **types** for negotiator i, and $\Theta = \Theta_1 \times \Theta_2$ be the set of all type profiles. We now also need to adjust the regularity assumption concerning the negotiators' preferences over bundles. Specifically, we need to modify the deal-breakers conditions since both issues can now potentially have unacceptable alternatives. For any type θ_i, we let $B(\theta_i) = \{(x, y) \in X \times Y \mid x \; \theta_i^X \; o_X \text{ and } y \; \theta_i^Y \; o_Y\}$ to be the set of all acceptable bundles.

Definition S.1. *Under the symmetric treatment of the outside options, a preference map Λ is regular if the following hold for all $i \in N$ and $\theta_i \in \Theta_i$:*

i. [Monotonicity] For any $x, x' \in \overline{X}$ and $y, y' \in \overline{Y}$ with $(x, y) \neq (x', y')$,

$$(x, y) \; P_i \; (x', y') \text{ for all } R_i \in \Lambda(\theta_i) \text{ whenever } x \; \theta_i^X \; x' \text{ and } y \; \theta_i^Y \; y'.$$

ii. [Consistency] For any $\theta_i' \in \Theta_i$ with $B(\theta_i) \subseteq B(\theta_i')$,

$$\Lambda(\theta_i')|_{B(\theta_i)} = \Lambda(\theta_i)|_{B(\theta_i)}.$$

iii. [Deal-breakers] $(o_X, o_Y) \; P_i \; (x, y)$ for all $R_i \in \Lambda(\theta_i)$ whenever $o_X \; \theta_i^X \; x$ or $o_Y \; \theta_i^Y \; y$.

Proposition 1. *Under the symmetric treatment of the outside options, there is no mediation rule f that is strategy-proof, individually rational, and efficient.*

Proof. Consider the preference profile $(\theta_1, \theta_2) = (\theta_1^{x_m}, \theta_1^{y_m}, \theta_2^{x_1}, \theta_2^{y_1})$. That is, both negotiators find all alternatives acceptable. Let $(x, y) = f(\theta_1, \theta_2)$. Because negotiators' preferences over alternatives are diametrically opposed for each single issue, there is at least one negotiator $i \in N$ and an issue for which negotiator i does not get her top alternative for that issue. Suppose, without loss of generality, that this negotiator is 1 and the issue is X: that is, $x \neq x_1$. Consider the new profile where only negotiator 1's preferences are different, $(\theta_1', \theta_2) = (\theta_1^{x_1}, \theta_1^{y_1}, \theta_2^{x_1}, \theta_2^{y_1})$. Individual rationality of f and deal-breakers property imply that $f(\theta_1', \theta_2) = (x_1, y_1)$ because all other bundles are unacceptable for type $(\theta_1^{x_1}, \theta_1^{y_1})$.

To conclude, we already know that $f(\theta_1, \theta_2) = (x, y)$ and $x \neq x_1$, which implies $x_1 \; \theta_1^{x_m} \; x$. Because y_1 is negotiator 1's best alternative in issue Y, either $y = y_1$ or $y_1 \; \theta_1^{y_1} \; y$ is true. In either case, Monotonicity and transitivity of preferences imply $(x_1, y_1) \; P_1 \; (x, y)$ for all $R_1 \in \Lambda(\theta_1)$. Thus, by misrepresenting her preferences at profile (θ_1, θ_2), negotiator 1 can achieve the bundle (x_1, y_1) that is strictly better than (x, y) for all $R_1 \in \Lambda(\theta_1)$, contradicting the strategy-proofness of f. □

Mediation with Continuum of Alternatives

We extend the characterization of the class of logrolling rules to a continuous analogue of our model.[3] Suppose now that the issues X and Y are two closed and convex intervals of the real line. The outside options, o_X and o_Y, may or may not be the elements of these sets. We assume, without loss of generality, that $X = Y = [0, 1]$, with the interpretation that the negotiators aim to divide a unit surplus in each issue. To keep the notation consistent with the main text, let a bundle $b = (x, y)$ indicate what negotiator 2 gets in the two issues, i.e., negotiator 2 gets $x \in X$ and $y \in Y$, and thus, negotiator 1 gets the remaining $1 - x$ and $1 - y$ in issues X and Y, respectively. Agents having diametrically opposed preferences on each issue means that for any issue $Z \in \{X, Y\}$ and two alternatives $z, z' \in Z$, negotiator 1 (respectively 2) prefers z to z' whenever $z < z'$ (respectively $z > z'$). The value/ranking of the outside option o_X in issue X is each negotiator's private information. However, the value/ranking of the outside option o_Y in issue Y is common knowledge, and both negotiators prefer all $y \in Y$ to o_Y.

For any $\ell \in [0, 1]$, type ℓ of negotiator 1 (respectively 2), denoted by θ_1^ℓ (respectively θ_2^ℓ), prefers the outside option o_X to all alternatives $k \in [0, 1]$ with $\ell < k$ (respectively $\ell > k$).[4] In parallel with the discrete case, we denote a mediation rule by $f = [f_{\ell,j}]_{(\ell,j) \in [0,1]^2}$ where $f_{\ell,j} = f(\theta_1^\ell, \theta_2^j)$ for all $0 \leq \ell, j \leq 1$.[5] The negotiators have no mutually acceptable

[3] Matsuo (1989) shows that it is possible to overcome the impossibility in the bilateral exchange model of Myerson and Satterthwaite (1983) by restricting to a finite number of types. This section also shows that the possibility results in our main model are not driven by the finiteness of the number of types.

[4] In other words, $1 - \ell$ (respectively ℓ) is the least acceptable amount of X for type θ_1^ℓ (respectively θ_2^ℓ). Therefore, all k with $\ell \geq k$ (respectively $\ell \leq k$) are deemed acceptable by type θ_1^ℓ of negotiator 1 (respectively, by type θ_2^ℓ of negotiator 2).

[5] We assume, without loss of generality, that each negotiator has at least one acceptable alternative. Therefore, there is no type profile where a negotiator deems all alternatives unacceptable.

alternative in issue X at type profile $(\theta_1^\ell, \theta_2^j)$ when $\ell < j$. The set of mutually acceptable alternatives in issue X is $A(\theta_1^\ell, \theta_2^j) = [j, \ell] = X_{j\ell}$ whenever $\ell \geq j$. We use Θ_i as the set of all types of negotiator i and $\theta_i \in \Theta_i$ as the generic element whenever there is no need to specify the type's least acceptable alternative. The regularity and quid pro quo conditions in the main text directly apply here. The same is true for the definitions of strategy-proofness, efficiency, and individual rationality. In this framework an injective and order-reversing function $t : X \to Y$ corresponds to a strictly decreasing function. When (X, d) is a metric space with a proper metric d, the connected set $X_{j\ell}$ with $\ell \geq j$ is a nonempty, compact, and convex subset of X. Each t function in Figure 1 (in fact any such decreasing function) generates a set of logrolling bundles, B^t.

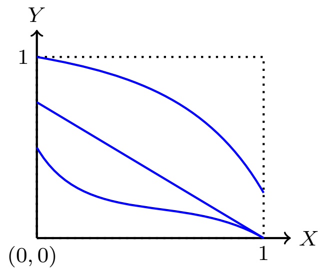

Figure 1: A possible set of logrolling bundles

Definition S.2. *A linear order $\succeq$ on X is said to be **quasi upper-semicontinuous over** $X_{j\ell}$ with $\ell \geq j$ if for all $x, x' \in X_{j\ell}$ with $x \neq x'$, $x \succeq x'$ implies that there exists an alternative $x'' \in X_{j\ell}$ and a neighborhood $\mathcal{N}(x')$ of x' such that $x'' \succeq x'''$ for all $x''' \in \mathcal{N}(x') \cap X_{j\ell}$.*[6]

The binary relation $\succeq$ is quasi upper-semicontinuous if it is quasi upper-semicontinuous over all compact subsets $X_{j\ell}$ of X. Theorem 1 of Tian and Zhoub (1995) proves that quasi upper-semicontinuity is both necessary and sufficient for $\succeq$ to attain its maximum over all compact subsets $X_{j\ell}$ of X. Therefore, the analogous characterization in the continuous model reads as follows.

Theorem S.1. *Suppose that the preference map Λ satisfies quid pro quo. The mediation rule f is strategy-proof, efficient, and individually rational if and only if there exists an alternative $y \in Y$, a strictly decreasing function $t : X \to Y$ and a linear order $\succeq_t$ on X, which are induced by the preference map Λ, such that $f = f^{\succeq_t}$; namely*

$$f_{\ell,j} = \begin{cases} \left(x^*_{X_{j\ell}}, t\left(x^*_{X_{j\ell}}\right)\right), & \text{if } j \leq \ell \\ (o_X, y), & \text{otherwise} \end{cases}$$

where $x^*_{X_{j\ell}} = \max\limits_{X_{j\ell}} \succeq_t$ is well-defined.

Analogous to the discrete case, we use the following continuously indexed matrix to describe a mediation rule $f^{\succeq_t}$.

[6] This is Definition 2 in Tian and Zhoub (1995).

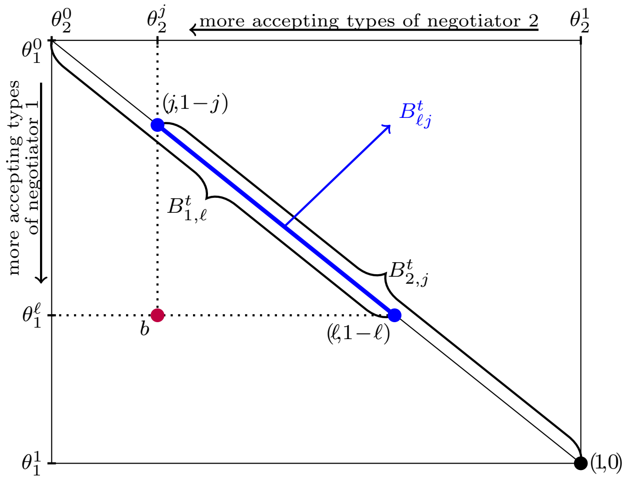

Figure 2: Adjacent rules in the continuous case when $t(x) = 1 - x$

The rows, i.e., the vertical axis, correspond to the types of negotiator 1 and columns, i.e., the horizontal axis, indicate all possible types of negotiator 2. Each point on the main diagonal represents a logrolling bundle for the mediation rule that is described in Theorem S.1, and each logrolling bundle appears only once on this diagonal. The bundle b, for example, represents the value of $f^{\succeq_t}$ when the types of negotiator 1 and 2 are θ_1^ℓ and θ_2^j, respectively. When the type profile is (θ_1^1, θ_2^1), negotiator 1 finds all alternatives acceptable and negotiator 2 deems all alternatives except 1 unacceptable.

If, for example, $t(x) = 1-x$, then the set of logrolling bundles is $B^t = \{(x, 1-x)|x \in [0,1]\}$. Then the only mutually acceptable logrolling bundle is $(1, 0)$ at type profile (θ_1^1, θ_2^1). The set of all acceptable logrolling bundles for type θ_1^ℓ of negotiator 1 is denoted by $B^t_{1,\ell}$, which consists of all the logrolling bundles on the upper portion of the main diagonal, starting from the top-left corner bundle, $(0, 1)$, and goes all the way down to the bundle $(\ell, 1-\ell)$. That is, $B^t_{1,\ell} = \{(k, 1-k) \in B^t \mid 0 \le k \le \ell\}$. Similarly, the set of all acceptable logrolling bundles for type θ_2^j of negotiator 2 is represented by $B^t_{2,j}$ and consists of all the bundles on the lower portion of the main diagonal, i.e., all bundles from $(j, 1-j)$ to $(1, 0)$. Namely, $B^t_{2,j} = \{(k, 1-k) \in B^t \mid j \le k \le 1\}$. Thus, the set of mutually acceptable logrolling bundles at the type profile $(\theta_1^\ell, \theta_2^j)$ is the intersection of these two sets, i.e., $B^t_{\ell j} = B^t_{1,\ell} \cap B^t_{2,j}$. Theorem S.1 says that bundle $b = f^{\succeq_t}_{\ell,j}$ is the logrolling bundle that maximizes $\succeq_t$ over the set $B^t_{\ell j}$ (see Figure 2). Such a maximal bundle is always unique because $\succeq_t$ is antisymmetric.

Proof of Theorem S.1:

Proof of 'if': The same arguments in the proof of Theorem 2 suffice to verify that the mediation rule described in Theorem S.1 is individually rational and efficient. Lemma 4 also holds in the continuous case. The proof of part (i) of Lemma 4 is straightforward; given the location of a logrolling bundle $a = (x_a, t(x_a))$ on the main diagonal, $f^{\succeq_t}_{\ell,j}$ can be a only if $a \in B^t_{\ell j}$, and so, a can never appear outside of its value region $V(a)$. To prove part (ii), let $f^{\succeq_t}_{\ell,j} = a$, $f_{s,r} = b = (x_b, t(x_b))$ and $a \neq b$. Also, suppose for a contradiction that $a, b \in V(a) \cap V(b)$. Therefore, we have $a, b \in B^t_{\ell j} \cap B^t_{sr}$. The alternative x_a beats x_b with respect to $\succeq_t$ because a wins over $B^t_{\ell j}$, i.e., $f^{\succeq_t}_{\ell,j} = a$. Likewise, x_b beats x_a with respect to $\succeq_t$ because b wins over B^t_{sr}, i.e., $f^{\succeq_t}_{s,r} = b$. The last two observations contradict with the assumption that $\succeq_t$ is antisymmetric. To prove part (iii), suppose that $f^{\succeq_t}_{\ell,s} = a$ and $f_{j,s} = b$ where $\ell < j$, whereas a appears below b on the main diagonal. This is possible only when $a, b \in V(a) \cap V(b)$, contradicting part (ii). Similar arguments prove the claim when bundles a and b are on the same row.

Now we prove that $f^{\succeq_t}$ is strategy-proof. It suffices to consider the deviations of one negotiator. Take any $\ell, j \in [0,1]$ such that $f(\theta_1^\ell, \theta_2^j) = f_{\ell,j} = (o_X, y)$ (see figure $3-a$). Deviating from θ_1^ℓ does not benefit negotiator 1 if he deviates to θ_1^s where $s < j$ because the outcome of $f^{\succeq_t}$ will not change. However, if negotiator 1 deviates to some $s \geq j$ and get some b, we know that b is one of the logrolling bundles in B^t_{sj}. However, all of the bundles in B^t_{sj} are unacceptable for type θ_1^ℓ of negotiator 1 since $\ell < s$, and so, not preferable to (o_X, y) by the deal-breaker property.

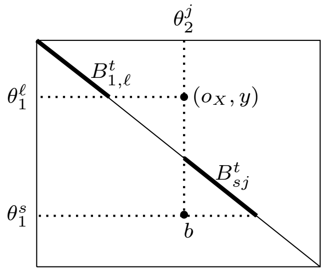

Figure 3-a

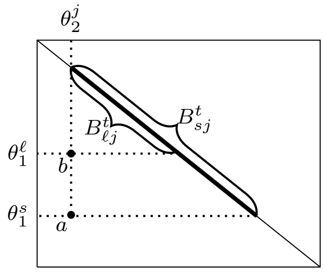

Figure 3-b

Now take any $\ell, j \in [0,1]$ such that $\ell \geq j$ and $f^{\succeq_t}_{\ell,j} = b \in B^t$ (see Figure $3-b$). Deviating from θ_1^ℓ does not benefit negotiator 1 if he deviates to θ_1^s where $s < j$ because the outcome of $f^{\succeq_t}$ would be (o_X, y), which is not better than b by the deal-breaker property. If negotiator 1 deviates to some $\ell > s \geq j$ and get some a, then a must appear above b on the main diagonal (part (ii) of Lemma 4). That means $x_a \; \theta_1^\ell \; x_b$. Moreover, we must have $x_b \;\; \succeq_t \;\; x_a$because b was chosen while both a and b were available. Therefore by quid pro quo, negotiator 1 must find b at least as good as a at all admissible preferences, and thus, deviating to s is not profitable.

Finally, suppose that negotiator 1 deviates to some $s > \ell \geq j$ and get some a (see figure 3-b). Therefore, x_a beats x_b with respect to $\succeq_t$ because both a and b are in B^t_{sj} and a is chosen. Thus, a cannot be an element of $B^t_{\ell j}$ as x_b is the maximizer of $\succeq_t$ over $X_{j\ell}$. Thus, $a \in B^t_{sj} \setminus B^t_{\ell j}$, implying that a is not acceptable for type θ_1^ℓ, and so, deviating to θ_1^s is not profitable by the deal-breaker property. Hence, $f^{\succeq_t}$ is strategy-proof.

Proof of 'only if': The same arguments in the proof of Theorem 1 suffice to show that there must exist some $y \in Y$ such that $f_{\ell j} = (o_X, y)$ for all $\ell, j \in [0,1]$ with $\ell < j$. Consider now for $\ell \geq j$.

It is easy to verify that WARP, i.e., Lemma 1 in the proof of Theorem 1, also holds here. That is, if $x, x' \in A(\theta) \cap A(\theta') \neq \emptyset$, $x \neq x'$ and $x = f^X_\theta$, then $f^X_{\theta'} \neq x'$.

Lemma S.1 (Existence of t). *There exists a strictly decreasing function $t : X \to Y$ such that $f_{k,k} = (k, t(k))$ for every $k \in [0, 1]$.*

Proof. We will prove that $f^Y_{\ell,\ell}\ \theta^Y_1\ f^Y_{k,k}$, i.e. $f^Y_{\ell,\ell} < f^Y_{k,k}$ for each $k \in [0, 1]$ and $\ell \in [0, 1]$ with $\ell > k$. If this statement is correct, then we have the desired result when we set $t(k) = f^Y_{k,k}$ for all k, including t being strict because θ^Y_1 is a transitive and irreflexive relation over Y.

Suppose for a contradiction that there exists some $k \in [0, 1]$ and $\ell \in [0, 1]$ with $\ell > k$ such that $f^Y_{\ell,\ell} \geq f^Y_{k,k}$. First consider $f_{\ell,k}$: Efficiency and individual rationality of f and regularity of preferences imply that $f^X_{\ell,k} \in [k, \ell]$, i.e., $f^X_{k,k} \leq f^X_{\ell,k} \leq f^X_{\ell,\ell}$. Now consider $f^Y_{\ell,k}$. If $f^Y_{\ell,k} < f^Y_{\ell,\ell}$, then by monotonicity θ^k_2 would deviate to θ^ℓ_2 because $f_{\ell,\ell}$ gives more to negotiator 2 in both issues than $f_{\ell,k}$. Thus, we must have $f^Y_{\ell,k} \geq f^Y_{\ell,\ell}$. If $f^Y_{\ell,k} > f^Y_{k,k}$, then by monotonicity θ^ℓ_1 would deviate to θ^k_1 because $f_{k,k}$ gives more to negotiator 1 in both issues than $f_{\ell,k}$. Thus, we must have $f^Y_{\ell,k} \leq f^Y_{k,k}$. But the last two inequalities imply that we have $f^Y_{\ell,\ell} \leq f^Y_{\ell,k} \leq f^Y_{k,k}$. Together with our presumption, the last inequality yields $f^Y_{\ell,\ell} = f^Y_{k,k} = f^Y_{\ell,k}$. But then because $\ell > k$, monotonicity implies that either negotiator θ^k_2 profitably deviates to θ^ℓ_2 to get $f_{\ell,\ell}$ or negotiator θ^ℓ_1 profitably deviates to θ^k_1 to get $f_{k,k}$, contradicting strategy-proofness of f. □

Therefore, a strategy-proof, efficient, and individually rational f implies a strictly decreasing function $t : X \to Y$ and a non-empty set of bundles $B^t = \{(x, t(x)) | x \in X\}$, which constitutes the set of bundles on the main (first) diagonal. For any $1 \leq j \leq \ell \leq m$ let $B^t_{j\ell} = \{(k, t(k)) \in B^t \mid j \leq k \leq \ell\}$ denote the bundles on the main diagonal between row j to ℓ. Similar to the proof of Theorem 1, we construct $\trianglerighteq$ as follows: Take any type profile $\theta = (\theta^\ell_1, \theta^j_2)$ where $1 \leq j \leq \ell \leq m$. We say $f^X_{\ell,j} \trianglerighteq x$ whenever $x \in X_{j\ell}$. WARP implies that $\trianglerighteq$ is antisymmetric and reflexive. However, it is not necessarily complete.

Similar to Lemma 3, one can verify that the binary relation $\trianglerighteq$ is transitive. Furthermore, for all $1 \leq j < \ell \leq m$, $f_{\ell,j} = \left(x^*_{X_{j\ell}}, t\left(x^*_{X_{j\ell}}\right)\right) \in B^t_{j\ell}$ where $x^*_{X_{j\ell}} = \boldsymbol{max}_{X_{j\ell}} \trianglerighteq$. Hence, $f = f^{\trianglerighteq}$.

By the Szpilrajn's extension theorem (Szpilrajn 1930), one can extend $\trianglerighteq$ to a complete order. This extension will clearly preserve the maximal elements in every compact subset $X_{j\ell}$ because the maximal elements in every set $X_{j\ell}$ already have a complete relation with all the elements in that set. Finally, Theorem 1 in Tian and Zhoub (1995) proves that quasi upper-semicontinuity is both necessary and sufficient for $\trianglerighteq$ to attain its maximum on all compact subsets $X_{j\ell}$, and so $\trianglerighteq$ must be quasi upper-semicontinuous.

Finally, we need to prove that $\trianglerighteq \in \Pi_\Lambda$, i.e., t and $\trianglerighteq$ are induced by the preference map Λ. To prove that $\trianglerighteq$ and t satisfy the first part of Definition 5, let $\ell, j \in [0, 1]$ be two distinct alternatives and $\ell \trianglerighteq j$. Suppose, without loss of generality, that $j\ \theta_1\ \ell$, namely $j < \ell$. Strategy-proofness of f and consistency of preferences require that $f_{\ell,j}\ R_1\ f_{j,j}$, or equivalently $(\ell, t(\ell))\ R_1\ (j, t(j))$ for all admissible $R_1 \in \Lambda(\theta_1)$ and $\theta_1 \in \Theta_1$ satisfying $j, \ell \in A(\theta_1)$, as required by part (i). To prove part (ii) suppose for a contradiction that there is some $y \in Y$ with $t(\ell)\ \theta^Y_1\ y\ \theta^Y_1\ t(j)$ such that (j, y) Pareto dominates

$(\ell, t(\ell)) = f_{\ell,j}$. Because both of these bundles are acceptable at the profile $(\theta_1^\ell, \theta_2^j)$, the existence of such bundle, i.e., (j, y), contradicts with the presumption that f is efficient.

We now prove that $\trianglerighteq$ and t satisfy the second part of Definition 5. First recall that all sets of the form $X_{j\ell}$ with $1 \leq j \leq \ell \leq m$ designate all the connected subsets of X. By the construction of $\trianglerighteq$ we already know that every doubleton $\{x, x'\} \subseteq X_{j\ell}$ has a least upper bound in $X_{j\ell}$, which is $x^*_{X_{j\ell}}$, and thus the poset $(S, \trianglerighteq)$ is a semilattice for all connected subset S of X. Hence, $\trianglerighteq \in \Pi_\Lambda$. ■

Modeling Conflicting Preferences

Let X be a nonempty set of available alternatives, and Θ be the set of all linear orders on X. Define $max(\theta)$ as the maximal element of the preference ordering $\theta \in \Theta$, namely if $x^\star = max(\theta)$, then $x^\star \ \theta \ x$ for all $x \in X \setminus \{x^\star\}$. Therefore, a ***two-person, single-issue dispute*** (dispute in short) problem is a list $D = (\theta_1, \theta_2, X)$ where $\theta_i \in \Theta$ for $i = 1, 2$ and $max(\theta_1) \neq max(\theta_2)$.

For any nonempty subset $\widetilde{X} \subseteq X$, let $\theta|_{\widetilde{X}}$ denote the restriction of the preference ordering $\theta \in \Theta$ on $\widetilde{X}$. Therefore, define $\widetilde{D} = (\widetilde{\theta}_1, \widetilde{\theta}_2, \widetilde{X})$ to be a dispute reduced from $D = (\theta_1, \theta_2, X)$ whenever $\widetilde{X} \subseteq X$ and $\widetilde{\theta}_i = \theta_i|_{\widetilde{X}}$ for $i = 1, 2$.

Proposition S.1. *By eliminating all the Pareto inefficient alternatives, any dispute D can be reduced to an equivalent dispute $\widetilde{D}$ where the negotiators' preferences are diametrically opposed.*

A similar result, which we omit for brevity, holds for two-person, multi-issue disputes whenever preferences over bundles satisfy monotonicity.[7]

Proof. Let $\widetilde{A} \subseteq A$ be the set of alternatives that survive the elimination of Pareto inefficient alternatives. That is, none of the alternatives in $\widetilde{A}$ is Pareto inefficient. Renumber the elements in $\widetilde{A}$, and suppose, without loss of generality, that $\widetilde{A} = \{x_1, ..., x_m\}$ where $m \geq 2$, and negotiator 1 ranks alternatives as $x_k \ \widetilde{\theta}_1 \ x_{k+1}$. If x_m is not the best alternative for $\widetilde{\theta}_2$ on $\widetilde{A}$, then there must exist some x_k where $k < m$ such that $x_k \ \widetilde{\theta}_2 \ x_m$. But this contradicts the assumption that x_m is not Pareto inefficient. Thus, negotiator 2 must rank x_m as the top alternative. With similar reasoning, if x_{m-1} is not negotiator 2's second-best alternative, then it must be Pareto inefficient, contradicting the assumption that x_{m-1} survives after the deletion of Pareto inefficient alternatives. Iterating this logic implies that the rankings of the negotiators must be diametrically opposed. □

[7]See Section 2 for the formal definition of monotonicity.

The Revelation Principle

A mediation mechanism with veto rights $\Gamma = \big(S_1, S_2, g(.)\big)$ is a collection of action sets (S_1, S_2) and an outcome function $g : S_1 \times S_2 \to X \times Y$. The mechanism Γ combined with possible types (Θ_1, Θ_2) and preferences over bundles (R_1, R_2) with $R_i \in \Lambda(\theta_i)$ for all i defines a game of incomplete information. A strategy for negotiator i in the game of incomplete information created by a mechanism Γ is a function $s_i : \Theta_i \to S_i$.

Lemma S.2 (Revelation Principle in Dominant Strategies). *Suppose that there exists a mechanism $\Gamma = \big(S_1, S_2, g(.)\big)$ that implements the mediation rule f in dominant strategies. Then f is strategy-proof and individually rational.*

Proof. If Γ implements f in dominant strategies, then there exists a profile of strategies $s^*(.) = \big(s_1^*(.), s_2^*(.)\big)$ such that $g\big(s^*(\theta)\big) = f(\theta)$ for all $\theta \in \Theta$, and for all $i \in I$ and all $\theta_i \in \Theta_i$,

$$g\big(s_i^*(\theta_i), s_{-i}(\theta_{-i})\big)\ R_i\ g\big(s_i'(\theta_i'), s_{-i}(\theta_{-i})\big) \tag{1}$$

for all $R_i \in \Lambda(\theta_i)$, $\theta_i' \in \Theta_i$, $\theta_{-i} \in \Theta_{-i}$ and all $s_i'(.), s_{-i}(.)$. Condition 1 must also hold for s^*, meaning that for all i and all $\theta_i \in \Theta_i$,

$$g\big(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})\big)\ R_i\ g\big(s_i^*(\theta_i'), s_{-i}^*(\theta_{-i})\big) \tag{2}$$

for all $R_i \in \Lambda(\theta_i)$, $\theta_i' \in \Theta_i$, and all $\theta_{-i} \in \Theta_{-i}$. Because $g(s^*(\theta)) = f(\theta)$ for all $\theta \in \Theta$, the last inequality implies that for all i and all $\theta_i \in \Theta_i$,

$$f(\theta_i, \theta_{-i})\ R_i\ f(\theta_i', \theta_{-i}) \tag{3}$$

for all $R_i \in \Lambda(\theta_i)$, $\theta_i' \in \Theta_i$, and all $\theta_{-i} \in \Theta_{-i}$.

Moreover, because mechanism Γ allows each negotiator to veto the proposed bundle and receive the outside options in each issue, there also exists a deviation strategy $\hat{s}_i(.)$ for any strategy $s_i(.)$ such that $g(\hat{s}_i(\theta_i), s_{-i}) = \big(o_X, o_Y\big)$ for all $\theta_i \in \Theta_i$ and all $s_{-i} \in S_{-i}$. The idea is that the negotiator i plays in $\hat{s}_i(.)$ exactly the same way in $s_i(.)$ (for all θ_i's) until the ratification stage and vetoes the proposed bundle.

Therefore, if $\hat{s}_i(.)$ is such a deviation strategy for $s_i^*(.)$, then condition 1 must also hold for $\hat{s}_i(.)$, implying that for all i and $\theta_i \in \Theta_i$,

$$g\big(s_i^*(\theta_i), s_{-i}^*(\theta_{-i})\big)\ R_i\ g\big(\hat{s}_i(\theta_i'), s_{-i}^*(\theta_{-i})\big) = \big(o_X, o_Y\big)$$

for all $R_i \in \Lambda(\theta_i)$, $\theta_i' \in \Theta_i$ and all $\theta_{-i} \in \Theta_{-i}$. Because $g(s^*(\theta)) = f(\theta)$ for all $\theta \in \Theta$, the last condition means that for all i and all $\theta_i \in \Theta_i$,

$$f(\theta_i, \theta_{-i})\ R_i\ \big(o_X, o_Y\big) \tag{4}$$

for all $R_i \in \Lambda(\theta_i)$, $\theta_i' \in \Theta_i$ and all $\theta_{-i} \in \Theta_{-i}$. Hence, conditions 3 and 4 imply that f is strategy-proof and individually rational.[8] □

[8]If negotiators were to approve or veto each issue separately, then we would have

The deal-breakers assumption provides tractability in our analysis. One might wonder what role this assumption plays in obtaining a possibility result. We provide an example to show that the deal-breakers property is not necessary for a possibility result. (Also see Example 3 in the main text.)

Example 1 (Possibility despite the failure of deal-breakers): Suppose that $m = 3$ and consider our model with monotonic preferences satisfying the strong form of quid pro quo assumed in Section 5. We expand this domain of preferences by adding preferences that satisfy the following rankings:

$\theta_1^{x_1}$	$\theta_1^{x_2}$	$\theta_1^{x_3}$	$\theta_2^{x_1}$	$\theta_2^{x_2}$	$\theta_2^{x_2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$[(x_1, y_3)]$	$[(x_2, y_2)]$	$[(x_3, y_1)]$	$[(x_1, y_3)]$	$[(x_2, y_2)]$	$[(o_X, y_1)]$
$[(o_X, y_1)]$	$[(o_X, y_1)]$	$\begin{bmatrix}(x_2, y_2)\\(x_1, y_3)\\(o_X, y_1)\end{bmatrix}$	$[(x_2, y_2)]$	$[(o_X, y_1)]$	(o_X, o_Y)
$\begin{bmatrix}(x_3, y_1)\\(o_X, y_2)\\(x_2, y_1)\end{bmatrix}$	(x_1, y_3)		(x_3, y_1)	(x_3, y_1)	$\begin{bmatrix}(x_1, y_3)\\(x_2, y_2)\end{bmatrix}$
	(x_3, y_1)		(o_X, y_3)		
	(o_X, y_2)	(o_X, o_Y)	$[(o_X, y_1)]$	$[(x_1 y_3)]$	
(o_X, o_Y)	(o_X, o_Y)		(o_X, o_Y)	(o_X, o_Y)	

While the above is one specific preference profile where the deal-breakers assumption is violated, one can include as many preference profiles of this form to our domain as long as the relative rankings of the bundles in the brackets are preserved. (The relative rankings of the bundles within the same brackets can be chosen arbitrarily.) It is easy to verify that negotiator 1-optimal rule is still strategy-proof, efficient, and individually rational.

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$f(\theta_i, \theta_{-i})\ R_i\ \left(f_X(\theta_i', \theta_{-i}), o_Y\right)$ and $f(\theta_i, \theta_{-i})\ R_i\ \left(o_X, f_Y(\theta_i', \theta_{-i})\right)$, where $f_Z(.)$ denotes the suggested alternative by f in issue Z, together with conditions 3 and 4.